Overview of the Some Methods of the Design of Finite Impulse Response Filters: their advantages and disadvantages

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***Abstract*— Several methods for designing of FIR filters have been considered. The advantages and disadvantages of each method have been estimated.**

Keywords—filtering; design techniques; Fourier series; polynomials

I. Introduction

Filtering process is one of the most important processes of Digital Signal Processing (DSP). Filtering is a process that will transform the spectrum of a signal according to some rule of correspondence. The devices which perform the filtering is called as *filters*. Filters can be classified on the basis of the many features, such as on the method of the processing of the signals (analog and digital), dependence of the outputs of filters (recursive and nonrecursive), impulse response (Finite Impulse-Response filters – FIR filters, Infinite Impulse-Response Filters – IIR filters) an so on, though all classifications are interrelated each other, i.e., one classification might include or to be included to another one. In this paper the analytical methods of the design of the digital FIR and IIR filters will be considered.

Advantages of FIR filters with compared to IIR filters are:

1. Possibility to have exact linear phase;
2. Always stable with finite-duration transients;
3. Design methods generally linear in filter parameters;
4. Flexibility in the choice of the frequency response.

Disadvantages of FIR filters are:

1. Much higher filter order for fixed specifications;
2. Typically much higher delay introduced;
3. Iterative design methods need for computer-aided techniques.

A digital signal is a signal that represents a sequence of discrete values.

The systems that process the digital signals are called as *discrete-time systems*.

A design is the process of obtaining through the use of mathematical principles a discrete-time system that would produce a desired output signal when a specified signal is applied at the input.

Design of any filter can be obtained into four steps:

Approximation – the process of generating a transfer function that would satisfy the desired specifications, which may concern the amplitude or phase response or even the time-domain response of the filter. Approximation methods can be classified as *direct* or *indirect*. In direct methods the discrete-time transfer function is generated directly in the z domain whereas in indirect methods it is derived from a continuous-time transfer function. Approximations can also be classified as *noniterative* or *iterative*. The former usually entail a set of formulas and transformations that yield designs of high precision with minimal computational effort. Iterative methods are based on optimization algorithms. In these methods an initial design is assumed and is progressively improved until a discrete-time transfer function is obtained that satisfies the prerequisite specifications.

Realization – or synthesis of a digital filter is the process of generating a digital-filter network or structure from the transfer function or some other characterization of the filter.

Study of arithmetic errors – investigation and mitigation of any errors, which can cause the digital filter to violate the required specifications (numerical imprecisions, modeling inaccuracies, component tolerances, unusual or unexpected nonlinear effects, and so on).

Implementation – simulation of the filter network on a general-purpose digital computer, workstation, DSP chip (software implementation) and conversion of the filter network into a dedicated piece of hardware (hardware implementation).

A digital filter is a system that will receive an input in the form of a discrete-time signal and produce an output again in the form of a discrete-time signal.

II. Design of FIR filters

FIR filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time. FIR filters have finite-duration impulse response and consequently contain no poles (only zeros) in the finite z-plane. The ability to have an exactly linear phase response is the one of the most important of FIR filters. Sometimes FIR filters are called as nonrecursive filters. FIR filters are designed by using *direct noniterative* or *iterative* methods. The approximation problem is solved by the finding suitable approximations to various idealized filter transfer functions.

The transfer function of FIR filter is:

$$H(z) = \sum_{i=0}^{N-1} h(n) z^{-n} \quad (1)$$

where $h(n)$ – impulse response of FIR filter. The transfer function of FIR is the polynomial of z^{-1}.

Many methods are applied for the design of FIR filters. They will be considered below.

III. Window Design Method

Window method is one of the oldest methods for designing of FIR filters with linear phase. In this method, the infinite-duration impulse response will defined by expanding the frequency response of an ideal filter in a Fourier Series and then this Fourier Series will be truncated and smoothed with the window function. Then Inverse Discrete Fast Fourier Transform is needed to get the coefficients from the smoothed frequency response. Direct truncation of the frequency response is impossible because of the Gibbs phenomenon. The papers [1], [2] are dedicated to this method.

In [1] a set of windows called as Kaiser windows were introduced which are very close to optimum. It was showed that by adjusting a parameter of the window, the sidelobes could be diminished at the cost of increased transition bandwidth.

In [2] the Dolph-Chebyshev window was proposed. Its parameters could be readily determined directly because it has good spectral properties.

Next window methods are used for designing FIR filters:

1) Kaiser window;

2) Generalized cosine windows (rectangular, Hamming, Blackman and Hanning);

3) Bartlett triangular window.

The main advantages of window method are:

1) Relative simplicity;

2) Ease of use.

The disadvantages of this method are follows:

1) Window method is usually applied for design of prototype filters like lowpass, highpass, bandpass and it is limited for using in speech and image processing;

2) This method is applicable only if desired frequency response specification is absolutely integrable;

3) The window method has very little design flexibility, it means in low pass filter design, the passband edge frequency generally cannot be specified exactly since the window smears the discontinuity in frequency;

4) The calculation of the Fourier Series is enough difficult.

IV. Design Based on the Frequency-Response Characteristics

For some applications, the desired frequency response is not given at all frequencies but rather at a number of discrete frequencies. For this case, the frequency sampling method provides a solution for the design of non-recursive filters. In this method, the desired frequency response will be sampled at the equispaced frequencies. These frequency samples are considered to be Discrete-Fourier Transform coefficients of the impulse response of the designed FIR filter and it is possible to derive an approximation to desired frequency response. The lowpass filter's samples and frequency responses is given per example in Fig.1.

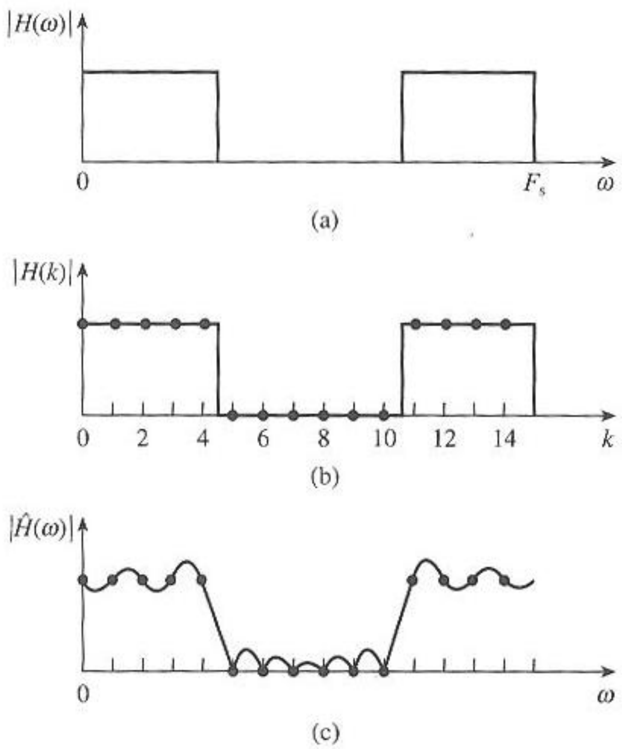

Fig. 1. (a) Frequency response of an ideal lowpass filter. (b) Samples of the ideal lowpass filter. (c) Frequency response of lowpass filter derived from the frequency samples of (b).

By using following Inverse Discrete Fourier Transform formula, the filter coefficients can be calculated.

$$h(n) = 1/N \sum_{n=0}^{N-1} H(k)\, e^{j\left(\frac{2\pi n}{N}\right)k} \quad (2)$$

Design of nonrecursive filters from frequency-response specifications has been considered by Martin [3], who specified initial values of the frequency response at selected frequencies, leaving unspecified values of the frequency response in preselected transition bands.

The frequency-sampling method has been applied to the design of low-pass and bandpass filters [5], [6].

In [5] the new approach was applied for direct searching of transition values of the sampled frequency response function for reduce the sidelobe level of the response. A search algorithm was derived and made it easier to obtain the experimental results.

In [6], the ideal frequency response of the filter was approximated by placing appropriate frequency samples in the *z*-plane and then choosing the remaining frequency samples to satisfy an optimization criterion. This technique had been applied successfully to the design of lowpass and bandpass filters, as well as wide-band differentiators.

The advantages of the frequency-sampling method are:

1) The mathematical solution of the optimization problem is considerably simple;

2) It leads to a recursive implementation of FIR filters what coefficients are integers, making the computation very fast even less precise;

3) Easy design of the filters with arbitrary magnitude and phase characteristics, or with any length impulse response;

4) Very useful for the design of non-prototype filters where the desired magnitude response can take any irregular shape.

The disadvantages are:

1) Because of the frequency response obtained by interpolation is equal to the desired frequency response only at the sampled points, and a finite error will be present at the other points, the results are not optimal;

2) It is necessary to program the linear optimization procedure, or to use some available optimization program and adapt it to the specific needs.

V. EQUIRIPPLE DESIGNES

This design technique is based on the solution of a system of nonlinear equations for generating a FIR filter with an equiripple approximation error. The idea of this method states that the unknown quantities are both the $(N + 1) / 2$ coefficients in the impulse response (we will assume that N is odd, and it is a symmetrical impulse response) and a set of $(N - 3)/2$ frequencies at which extrema of the approximation error occur. We will write constraint equations on the extrema and on the derivatives then we can obtain a system of $(N - 1)$ nonlinear equations in $(N - 1)$ unknowns. Many types of standard nonlinear optimization techniques can be used to solve these equations.

This method is divided into some submethods. As mentioned above, the basic idea of this method, consequently each of submethods is to design the filter coefficients again and again until a particular error is minimized. These submethods are described below:

1. In Weighted Chebyshev Approximation Method an error function is formulated for the desired filter in terms of a linear combination of cosine functions and is then minimized by using a very efficient multivariable optimization algorithm known as the Remez exchange algorithm. When convergence is achieved, the error function becomes equiripple as in other types of Chebyshev solutions. The amplitude of the error in different frequency bands of interest is controlled by applying weighting to the error function. The scientists Herrmann [7], Hofstetter and the others [8], Parks and McClellan [9], Rabiner and the others [10] worked under this method.
2. In Least Squared Error Frequency Domain Design Method the output samples are least mean-square estimates of the samples of some desired signal. The method requires that one know the input autocorrelation function and the cross- correlation function between the input and the desired output.
3. Nonlinear equation solution for maximal ripple FIR filters method for designing FIR filters solves a system of nonlinear equations to generate a filter with an equiripple approximation error.
4. Polynomial interpolation solution for maximal ripple FIR filters method is basically an iterative technique for producing a polynomial that has extrema of desired values. The algorithm begins by making an initial estimate of the frequencies at which the extrema of polinomial will occur and then uses the well-known Lagrange interpolation formula to obtain a polynomial that alternatively goes through the maximum allowable ripple values at these frequencies.

Advantages of equirripple design are:

1) It is possible to produce the lowest possible number of filter coefficients that just meets the requirement;

2) It gives the optimum transition band for the FIR filter.

The disadvantages are:

1) In order to use the equiripple design techniques, the user must program the nonlinear optimization procedure, or use some available routine and again adapt it to his needs;

2) Equiripple process is very time consuming;

3) One has to calculate a new filter according to some dynamically changing parameters;

4) There is no guarantee that the Remez Exchange algorithm will converge - it may converge to a false result (hence equiripple designs should always be checked): or it may not converge ever (resulting in hung computers, divide by zero errors and all sorts of other horrors).

VI. Analytical design method

In this method some polynomial will be chosen and then the some parameters of this polynomial will be transformed for becoming the transfer function of the FIR filter.

There exists a group of methods attributed to the analytic design procedures. The first submethod is based on Bernstein polynomials developed by Herrmann [11], Rajagopal and Dutta Roy [12], Cooklev and Nishihara [13].

The analytic design procedure is based on the first derivative of the pseudoamplitude *Q(w)* of an FIR filter was considered by Vlček, Zahradnik and Unbehauen [14]. They called this derivative the generating function.

One of the applied techniques is based on Chebyshev polinomials. Many people considered it [15], [116].

The application of that class of polynomials in filter theory had to wait until 1970, when Levy studied odd Achieser–Zolotarev polynomials with the application to the quasilowpass filters [17]. The FIR filter design based on Zolotarev polynomials was deeply studied by Chen and Parks [18] and Vlček and Unbehauen [19].

The advantages of this method are:

1) This design process is ten or hundred times faster than indirect methods (per example, equiripple method);

2) Easy to understanding and applying;

3) Lowest possible number of filter coefficients;

4) Optimal transition band.

The disadvantages are:

1) It is necessary to know and calculate some special functions such as theta function, eta function and etc.

Conclusions

The main design methods of FIR filters were described. The advantages and disadvantages were observed. The choice of the some design method is effective when one will be mean this advantages and disadvantages.

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