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Modeling and Electrical Characterization of Josephson Junction Using Approximation in the sense of Least Squares

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***Abstract*—In order to be able to realize out the mixing detection or harmonic generation functions, a non-linear circuit is necessary for different existing devices and for performing these types of operation, in the submillimetric and / or far-infrared domains (10 µm ≤ λ ≤ 1 mm), the spectral margin covered by this radiation ranging from 300 GHz to 30 THz. In these frequency domains, non-linear point devices are often used, unlike the optical domain where massive devices are widely used, among them the Josephson Junction (JJ) is mainly used in the case where low noise is desired. This paper present electrical characteristic of Josephson Junction (JJ) using Approximation in the sense of Least Squares, for different value of C_j, T, R_j.**

***Keywords*—Superconductor, Cooper Pair, Josephson Junction (JJ), Tunnel Effect, Least Squares Approximation.**

I. Introduction

Figure 1.a illustrates the basic structure of one Josephson junction in a large series array; the junction is an overlap between two superconductor's thin films that are separated by a thin oxide barrier. The Josephson Effect [1,2] is the remarkable effects of superconductivity, a macroscopic quantum phenomenon that appears at very low temperatures in some metals. In the superconductor state, the electrons attract two by two and form pairs, called Cooper Pairs [3]. The Josephson Effect is associated with the passage of these pairs by tunnel effect [4], through an insulating barrier placed between two superconductors, called Josephson Junction (fig. 1.b) (Superconductor-Insulator-Superconductor junction "S-I-S Josephson Junction").

The following figure (fig. 2) shows, energy diagram of a superconducting tunnel junction. The vertical axis is energy, and the horizontal axis shows the density of states. Cooper Pairs exist at the Fermi Energy, indicated by the dashed lines. A bias voltage V is applied across the junction, shifting the Fermi Energies of the two superconductors relative to each other by an energy eV, where e is the electron charge. Quasiparticle states exist for energies greater than Δ from the Fermi Energy, where Δ is the superconducting energy gap. Green and blue indicate empty and filled quasiparticle states, respectively, at zero temperature. This paper describes the electrical characterization for the Josephson Junction (S-I-S) using Approximation in the sense of Least Squares, for different value of C_j, T, R_j.

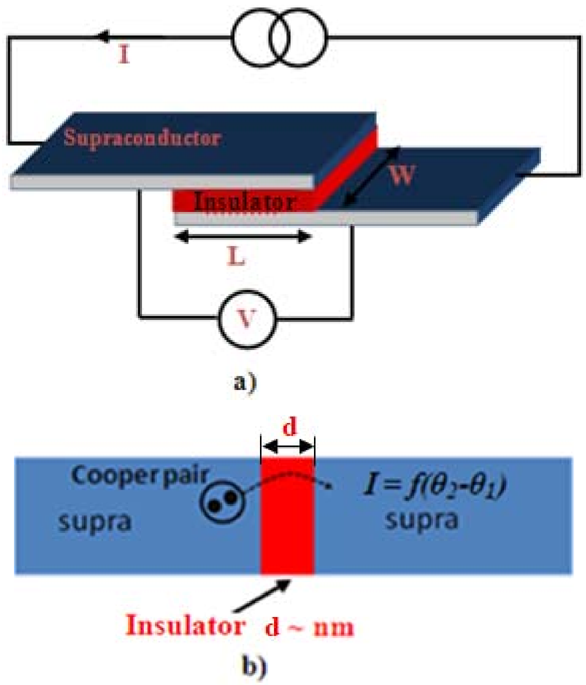

Fig. 1. Basic structure of Josephson Junction (JJ).

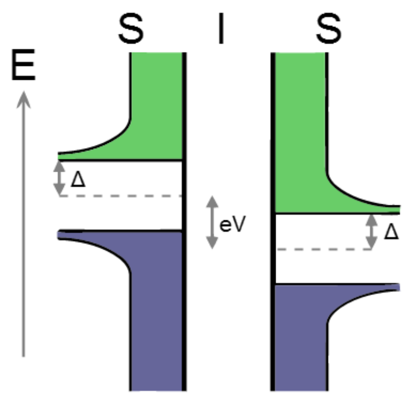

Fig. 2. Energy diagram of superconducting tunnel junction.

From figure 3, Current-Voltage (I-V) characteristics of superconducting tunnel junction, the Cooper Pair tunneling current is seen at V = 0, while the quasiparticle tunneling current is seen for V > 2Δ/e and V < -2Δ/e.

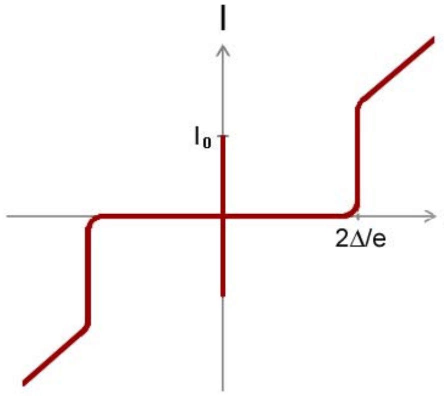

Fig. 3. Current-Voltage (I-V) characteristics of superconducting tunnel junction.

II. JOSEPHSON EFFECT

There are two types of Josephson Effects [5,6,7], the D.C. Josephson Effect and the A.C. Josephson Effect. These two effects were predicted by Brian David Josephson in 1962 from the BCS theory [8].

A. *DC Josephson Effect*

In the absence of voltage applied to the terminals of such a junction (fig. 4), a direct current, current of Cooper Pairs I_J, flows in the junction up to a critical value I_c or I_0, which depends on the geometry, the temperature and magnetic field.

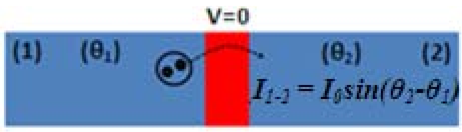

Fig. 4. D.C Josephson Effect: in the absence of potential difference, a current flows from the block (1) to the block (2).

Let's put ourselves in the case where V = 0, without potential difference between the two blocks. The number of particles passing from 1 to 2 during the unit of time, that is to say the flow of 1 to 2 corresponds to the decrease of the number of particles in box 1 and its increase in box 2 is:

$$I_{1\to 2} = \frac{dn_1}{dt} = \frac{dn_2}{dt} = I_0 \sin(\theta) \quad (1)$$

With $\theta = \theta_2 - \theta_1$

$$I_{1\to 2} = I_0 \sin(\theta_2 - \theta_1) \quad (2)$$

This current depends on the phase difference between the wave functions of the 2 condensates. It flows in the absence of potential difference, which is called continuous Josephson current [9,10,11].

It should be understood that I_0 is the maximum current that can flow in superconducting regime and in the absence of potential difference between the blocks.

B. *AC Josephson Effect*

Introduce a potential difference V different from 0 between the two superconductor's blocks (fig. 5).

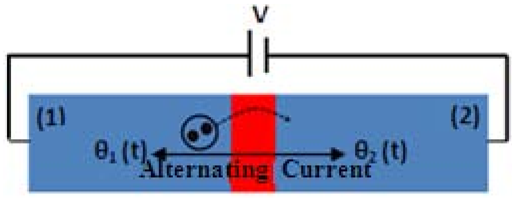

Fig. 5. A.C Josephson Effect in the presence of a D.C voltage, an alternating current crosses the junction

The intensity of the Josephson current remains:

$$I_{1\to 2} = I_0 \sin(\theta_2 - \theta_1) \quad (3)$$

But now the phase difference obeys the equation:

$$\frac{d}{dt}(\theta_2 - \theta_1) = \frac{qV}{\hbar} \quad (4)$$

And varies linearly with time:

$$(\theta_2 - \theta_1)(t) = \frac{qV}{\hbar} t + (\theta_2 - \theta_1)(0) \quad (5)$$

And the Josephson current is:

$$I = I_0 \sin\left(\frac{qV}{\hbar} t + (\theta_2 - \theta_1)(0)\right) \quad (6)$$

Is an alternating current of pulsation:

$$\omega = \frac{qV}{\hbar} \quad (7)$$

With $\hbar$ is Planck's constant and q is electron charge.

III. INSRTING A JOSEPHSON JUNCTION INTO CIRCUIT

For practical applications of devices having the Josephson Effect, it is important to have an equivalent circuit diagram that can represent them in a circuit. The commonly used basic model is known as RCSJ (fig. 6) (Resistively and Capacitively Shunted Junction) [12,13].

This model, represented in figure 6, involves an ideal junction JJ, which is a current source, in parallel with a capacitor C and a resistor R. the junction JJ is characterized by the relations (1) and (2).

The value of C depends on the junction dimensions and the permittivity of the insulation forming the barrier (in the case of a SIS structure), finally R is a function R (V) of the potential difference V across the junction, and this function is determined by the tunnel characteristic of the junction [12,13].

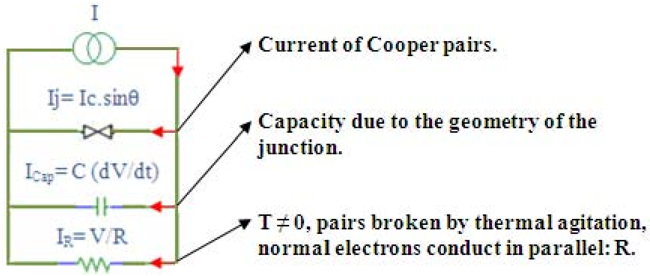

Fig. 6. RCSJ electrical schematic of a Josephson Junction with an ideal junction (JJ) with parallel capacitor and resistor.

This equivalent electrical schematic leads to an equation connecting the voltage to the current, which is in the form:

$$I_t = I_j + C\frac{dV(t)}{dt} + \frac{V(t)}{R(V)} \tag{8}$$

Given the relations (1) and (2), linking the phase difference to the current and the voltage in a Josephson Junction, we obtain:

$$I(t) = I_0 sin\theta + \frac{\hbar}{2q}C\frac{d^2\theta}{dt^2} + \frac{\hbar}{2qR(\theta)dt}\frac{d\theta}{dt} \tag{9}$$

It is impossible to find a general solution in an analytical form to this strongly non-linear equation, which must therefore be solved by numerical methods on a case-by-case basis. However, the formal identity between the equation I (t) and the mechanical equation of the movement of a pendulum made it possible to visualize the limit behavior of a Josephson junction by analogy with the mechanical behavior of the pendulum [12,13].

IV. APPROXIMATION IN THE SENSE OF LEAST SQUARES

A simple way to approach a function *f* by a sequence of polynomials over p_n an interval *I* is to use the Lagrange interpolation. However, there is not necessarily convergence of the sequence p_n towards the original function when the degree of polynomials tends towards infinity. The solution is to fix the degree and perform a piecewise interpolation (there is then convergence of p_n to *f* when the size of the sub-intervals tends towards 0) [14,15].

However, we prefer to use a representation on the whole interval *I*. We then adopt a different point of view of the interpolation: rather than requiring that p_n and *f* coincide in certain nodes, we ask that they be close in a more global way. Precisely, let *f* be a continuous function on the interval *I*, we will be interested in the two following cases [14,15].

- Continuous norm: We search for a polynomial p_n of degree inferior or equal to n such that the quantity (equation 10) is minimal.

$$\int_I |f(x) - p_n(x)|^2 dx \tag{10}$$

- Discrete norm: We search for a polynomial p_n of degree inferior or equal to n such that the quantity (equation 11) is minimal.

$$\sum_{i=0}^{N} |f(x_i) - p_n(x_i)|^2 \tag{11}$$

Or x_i ($0 \leq i \leq N$) is a subdivision of *I*, and $N \geq n$.

V. RESULTS AND DISCUSSION

Josephson Junction (S-I-S Josephson Junction) model is implemented under Matlab m-file (program code), to plot different curves, from detailed model in section 2 and 3. Our objective here is to implement the Approximation in the sense of Least Squares in Matlab, to identify different I-V characteristics of the Josephson Junction (JJ). This model can be used to simulate the Josephson Junction (JJ) response to different parameters related to the manufacturing technology.

Figure 7 shows the error in norm descrete according to degree n between the analytical model and the numerical model. Figures 8, 9 and 10 show a comparison between the results predicted by the Approximation in the sense of Least Squares (numerical model) with those calculated by the analytical model. From figures 7, 8, 9 and 10, it can be observed that good agreement between the numerical and analytical model.

Figures 8, 9 and 10 show the I-V characteristics of a single junction for different value of the temperature T, capacitor C_j and resistor R_j, which gives no linear access to a Josephson Junction. Between V = 0 and V = 2Δ/e the current is stabilized at 0. From 2Δ/e a new phenomenon occurs it is the passage of individual electron from broken pairs. Beyond 2Δ/e, the tunnel current is no longer that of Cooper Pairs, but that of individual electrons called quasi-particles. 2Δ : the energy that must be supplied to a pair of Cooper to break it.

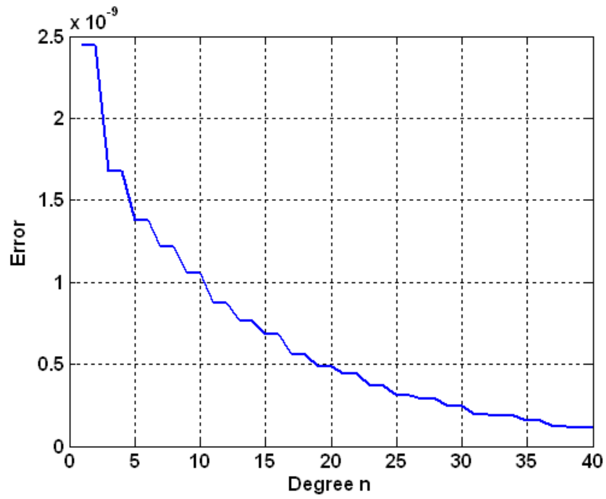

Fig. 7. Error in descrete norm according to degree n.

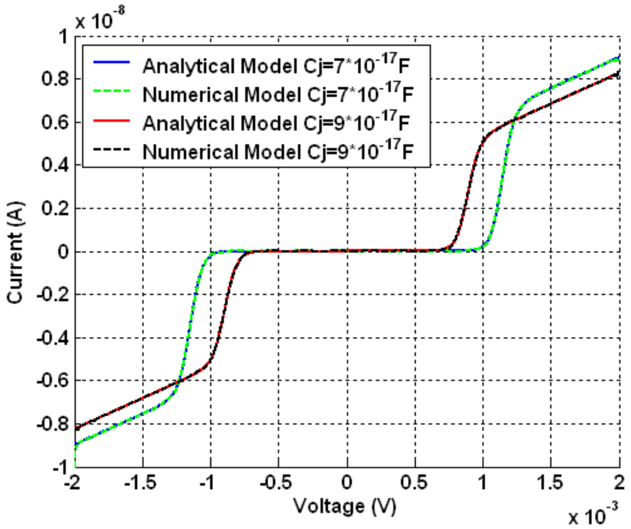

Fig. 8. Current-Voltage characteristics at T = 0.5 K, Rj = 350 KΩ, degree n=40.

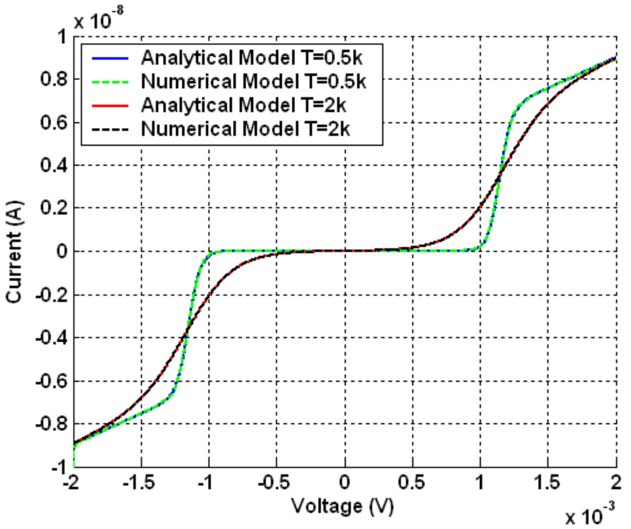

Fig. 9. Current-Voltage characteristics at Cj = 7e-17 F, Rj = 350 KΩ, degree n=40.

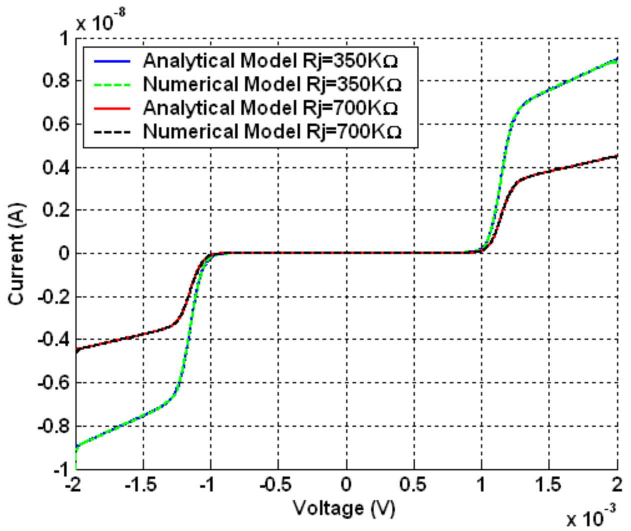

Fig. 10. Current-Voltage characteristics at T = 0.5 K, Cj = 7e-17 F, degree n=40.

VI. CONCLUSION

Josephson Junction (JJ) is the important components of the nonlinear superconducting electronics. The rise in frequency is limited on one side by the energy gap of the materials and on the other hand by the capacitive effect of the junction. In physics, the Josephson Effect is manifested by the appearance of a current between two superconductor's materials separated by a layer made of an insulating or non-superconductor metallic material. In the first case, the junction called "S-I-S Josephson junction" (Superconductor-Insulator-Superconductor) and in the second of "S-M-S junction" (Superconductor-Metal-Superconductor). In this work, we have presented electrical characteristics of Josephson Junction (JJ) using Approximation in the sense of Least Squares. A numerical approach based on an Approximation in the sense of Least Squares was developed and implemented in MATLAB, in the case of S-I-S Josephson Junction (Superconductor-Insulator-Superconductor) for different value of Cj, T, Rj. The S-I-S Josephson Junctions show high performance characteristics.

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